UNIT-5

ELECTROMAGNETIC WAVES-II

2 (9)

5.1

→ When an electromagnetic waves propagate through space there is a transfer of energy from the source to the destination.

Ex: Radio waves.

→ These waves bring power from transmitter to receiver and drive the first amplifier stage of the receiver.

→ The energy stored in an electric field and magnetic field is transmitted at a certain rate of energy flow which can be calculated with the help of Poynting theorem.

→ In order to find the power flow associated with an EM wave, it is necessary to develope a power theorem for electromagnetic fields is known as Poynting theorem.

<u>Poynting theorem:</u> states that "the net power flowing out of a given volume 'v' is equal to the time rate of decrease in the energy stored within volume 'v' minus the conduction losses."

It is mathematically expressed as

$$\boxed{\oint_S (E\times H)\cdot ds = -\frac{d}{dt}\int_v \left(\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right) dv - \int_v \sigma E^2\, dv}$$

Proof:

We know the vector identity $\nabla\cdot(A\times B) = B\cdot(\nabla\times A) - A\cdot(\nabla\times B)$

→ from that vector identity,

$$\nabla\cdot(E\times H) = H\cdot(\nabla\times E) - E\cdot(\nabla\times H)$$

$$= H\cdot\left(-\frac{\partial B}{\partial t}\right) - E\cdot\left(J + \frac{\partial D}{\partial t}\right)$$

$$= H\left(-\mu\frac{\partial H}{\partial t}\right) - E\left(\sigma E + \epsilon\frac{\partial E}{\partial t}\right)$$

$$\nabla\cdot(E\times H) = -\mu H\frac{\partial H}{\partial t} - E\sigma E - \epsilon E\frac{\partial E}{\partial t}$$

$$\nabla\cdot(E\times H) = -\epsilon\left[E\frac{\partial E}{\partial t}\right] - \mu\left[H\frac{\partial H}{\partial t}\right] - \sigma E^2 \longrightarrow (1)$$

Now $\frac{\partial}{\partial t}(E^2) = 2E\frac{\partial E}{\partial t} \Rightarrow E\frac{\partial E}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}E^2$

Similarly $\frac{\partial}{\partial t}(H^2) = 2H\frac{\partial H}{\partial t} \Rightarrow H\frac{\partial H}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}H^2$ $\longrightarrow (2)$

~~Substitute eq (2) in eq (1).~~

$$\nabla\cdot(E\times H) = -\epsilon\frac{1}{2}\frac{\partial}{\partial t}E^2 - \mu\frac{1}{2}\frac{\partial}{\partial t}H^2 - \sigma E^2$$

$$\nabla\cdot(E\times H) = -\frac{\partial}{\partial t}\left[\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right] - \sigma E^2$$

Apply volume integrals on both sides.

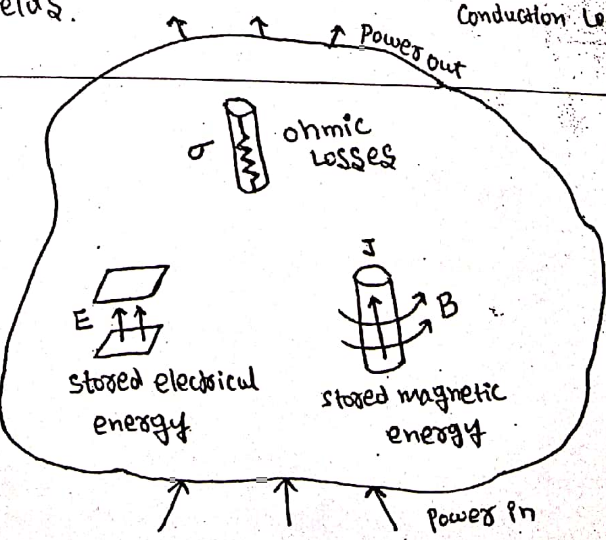
$$\int_v \nabla\cdot(E\times H)\,dv = -\frac{\partial}{\partial t}\int_v\left[\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right]dv - \int_v \sigma E^2\,dv$$

Applying the divergence theorem to the left-hand side gives

$$\boxed{\oint_S (E\times H)\cdot dS = -\frac{\partial}{\partial t}\int_v\left[\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right]dv - \int_v \sigma E^2\,dv} \longrightarrow (3)$$

Total Power Leaving the volume = Rate of decrease in energy stored in electric & magnetic fields − Ohmic Power dissipated (or) Conduction Losses.

→ The Poynting theorem is based on law of conservation of energy in electromagnetism.



Poynting Vector :

→ The quantity $E \times H$ on the left hand side of eq(3) is known as Poynting vector P. i.e $\boxed{P = E \times H}$

→ Power density associated with an electromagnetic wave [EM wave] is called Poynting vector.

→ Poynting vector is defined as the cross product of the vectors E and H.

→ Poynting vector gives Power per unit area. It is denoted with P and measured in watts/m^2

$$\boxed{\therefore P = E \times H} \quad (\text{or}) \quad \boxed{P(z,t) = E(z,t) \times H(z,t)}$$

Analysis: Expression for Poynting vector of EM wave in free space.

$$E = E_x a_x \quad \text{and} \quad H = H_y a_y$$

then $P = E \times H = E_x a_x \times H_y a_y = E_x H_y (a_x \times a_y) = E_x H_y a_z = P_z a_z$

$$\therefore P = P_z a_z$$

→ The above equation indicates that E, H and P are mutually perpendicular to each other.

→ Consider that Electric field propagates in free space given by

$$E = E_o e^{-\alpha z} \cos(\omega t - \beta z) a_x = E_o \cos(\omega t - \beta z) a_x \qquad \because \alpha = 0.$$

→ Similarly magnetic field

$$H = H_o \cos(\omega t - \beta z) a_y = \frac{E_o}{|\eta|} \cos(\omega t - \beta z) a_y \qquad \because \eta = \frac{E_o}{H_o} \Rightarrow H_o = \frac{E}{\eta}, \quad \eta = |\eta| \angle \theta$$

→ According to the Poynting theorem

$$P = E \times H = [E_o \cos(\omega t - \beta z) a_x] \times \left[\frac{E_o}{|\eta|} \cos(\omega t - \beta z) a_y\right]$$

$$P = \frac{E_o^2}{|\eta|} \cos^2(\omega t - \beta z)(a_x \times a_y) = \frac{E_o^2}{|\eta|} \cos^2(\omega t - \beta z) a_z$$

$$\boxed{\therefore P = \frac{E_o^2}{|\eta|} \cos^2(\omega t - \beta z) a_z} \quad W/m^2$$

This is nothing but the power density measured in watt/m^2.

→ Thus the Power passing particular area is given by

$\boxed{\text{Power} = \text{Power density} \times \text{area}}$ $\quad$ $\boxed{\text{Power density} \rightarrow \text{Poynting vector}}$

Time average Power (or) Average Power density (P_{avg}) :

Average Power of Poynting vector $P(z,t)$ over a one cycle length T is called time average power of an Electromagnetic wave. It is the function of z only and it is denoted with $P_{avg}(z)$.

$$\boxed{\therefore P_{avg}(z) = \frac{1}{T}\int_0^T P(z,t)\,dt}$$

Ex1 Find time average Power of an EM wave in free space.

sol In free space $P(z,t) = E \times H = \frac{E_o^2}{|\eta|}\cos^2(\omega t-\beta z)\,a_z$ w/m.

$$P_{avg} = \frac{1}{T}\int_0^T \frac{E_o^2}{|\eta|}\cos^2(\omega t-\beta z)\,dt = \frac{E_o^2}{|\eta|T}\int_0^T \frac{1+\cos 2(\omega t-\beta z)}{2}\,dt$$

$$P_{avg} = \frac{E_o^2}{T|\eta|}\left[\frac{t}{2} + \frac{\sin 2(\omega t-\beta z)}{2\cdot 2\omega}\right]_0^T = \frac{E_o^2}{T|\eta|}\left[\frac{t}{2} + \frac{\sin 2(\omega t-\beta z)}{4\omega}\right]_0^T$$

$$P_{avg} = \frac{E_o^2}{T|\eta|}\left[\frac{t}{2} + \frac{\sin(2\omega t-2\beta z)}{4\omega}\right]_0^T = \frac{E_o^2}{T|\eta|}\left[\frac{T}{2} + \frac{\sin(2\omega T-2\beta z)}{4\omega} - \frac{\sin(-2\beta z)}{4\omega}\right]$$

But $\omega T = 2\pi f \frac{1}{f} = 2\pi$, then

$$P_{avg} = \frac{E_o^2}{T|\eta|}\left[\frac{T}{2} + \frac{\sin(4\pi-2\beta z)}{4\omega} + \frac{\sin 2\beta z}{4\omega}\right]$$

$$P_{avg} = \frac{E_o^2}{T|\eta|}\left[\frac{T}{2} - \frac{\sin 2\beta z}{4\omega} + \frac{\sin 2\beta z}{4\omega}\right] = \frac{E_o^2}{T|\eta|}\times\frac{T}{2} = \frac{E_o^2}{2|\eta|}$$

$$\boxed{\therefore P_{avg} = \frac{E_o^2}{2|\eta|}\ \text{W/m}^2}$$

2. Find time average Power of an electromagnetic wave in a lossy medium.

Sol under lossy region,

$$E(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_x$$

$$H(z,t) = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_n)\, a_y$$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$\sin(0-\theta) = -\sin\theta$

$\sin(4\pi - \theta) = -\sin\theta$

Pointing vector $P(z,t) = E(z,t) \times H(z,t)$.

$$P(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_x \times \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, a_y$$

$$P(z,t) = \frac{E_0^2}{|\eta|} e^{-2\alpha z} \cos(\omega t - \beta z) \cos(\omega t - \beta z - \theta_n)\, a_z$$

$$\therefore P_{avg} = \frac{1}{T} \int_0^T P(z,t)\, dt$$

$$= \frac{1}{T} \int_0^T \frac{E_0^2}{|\eta|} e^{-2\alpha z} \frac{2 \cos(\omega t - \beta z) \cos(\omega t - \beta z - \theta_n)}{2}\, a_z .\, dt$$

$$= \frac{E_0^2 e^{-2\alpha z}}{2T|\eta|} \int_0^T \left[\cos(2\omega t - 2\beta z - \theta_n) + \cos\theta_n\right] a_z .\, dt$$

$$= \frac{E_0^2 e^{-2\alpha z}}{2T|\eta|} \left[\frac{\sin(2\omega t - 2\beta z - \theta_n)}{2\omega} + t \cos\theta_n\right]_0^T a_z .$$

$$= \frac{E_0^2 e^{-2\alpha z}}{2T|\eta|} \left[\frac{\sin(2\omega T - 2\beta z - \theta_n) - \sin(0 - 2\beta z - \theta_n)}{2\omega} + T\cos\theta_n\right] a_z$$

let $\omega T = 2\pi f T = 2\pi$ in above equation.

$$\therefore P_{avg} = \frac{E_0^2 e^{-2\alpha z}}{2T|\eta|} \left[\frac{\sin(4\pi - 2\beta z - \theta_n) + \sin(2\beta z + \theta_n)}{2\omega} + T\cos\theta_n\right] a_z$$

$$P_{avg} = \frac{E_0^2 e^{-2\alpha z}}{2T|\eta|} \left[-\sin(2\beta z + \theta_n) + \sin(2\beta z + \theta_n) + T\cos\theta_n\right] a_z$$

$$P_{avg} = \frac{E_0^2 e^{-2\alpha z}}{2T|\eta|} \left[T\cos\theta_n\right] a_z = \boxed{\frac{E_0^2 e^{-2\alpha z}}{2|\eta|} \cos\theta_n\, a_z = P_{avg}}$$

→ for lossless & free space region $\alpha = 0$ & $\theta = 0$

$$\therefore P_{avg} = \frac{E_0^2}{2|\eta|} a_z .$$

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Power loss in a plane conductor :

When an electromagnetic wave is propagating through the conducting medium, then the power ~~flow per unit area~~ (or) time average power is called power loss (or) power loss in a plane conductor.

→ Power loss in a plane conductor is

$$P_{(z,t)} \text{ (or) } P_{avg} = \frac{E_0^2}{2|\eta|} e^{-2\alpha z} \cos\theta_n$$

In conducting media $\theta_n = \frac{\pi}{4}$ and from skin depth $\alpha = \frac{1}{\delta}$

→ Intrinsic impedance

$$\eta = |\eta| \angle\theta_n = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{j}\sqrt{\frac{\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \angle \pi/4$$

$$\therefore |\eta| = \sqrt{\frac{\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma} \times \frac{2}{2} \times \frac{\sigma}{\sigma}} = \sqrt{\frac{2\omega\mu\sigma}{2\sigma^2}}$$

$$|\eta| = \frac{\sqrt{2}}{\sigma}\sqrt{\frac{\omega\mu\sigma}{2}} = \frac{\sqrt{2}}{\sigma}\alpha = \frac{\sqrt{2}}{\sigma\delta}$$

$$\boxed{\therefore |\eta| = \frac{\sqrt{2}}{\sigma\delta}}$$

$$\sqrt{j} = \frac{1+j}{\sqrt{2}} = \frac{\pi}{4} = 45^\circ$$

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

→ Power loss in a plane conductor is

$$P_{(z,t)} \text{ (or) } P_{avg} = \frac{E_0^2}{2\frac{\sqrt{2}}{\sigma\delta}} e^{-2\alpha z} \cos(45^\circ)$$

$$= \frac{E_0^2\sigma\delta}{2\sqrt{2}} . e^{-2z/\delta} . \frac{1}{\sqrt{2}}$$

$$= \frac{E_0^2\sigma\delta}{4} e^{-2z/\delta}$$

$$\boxed{\therefore P_{(z,t)} \text{ (or) } P_{avg} = \frac{E_0^2\sigma\delta}{4} e^{-2z/\delta}}$$

Surface Impedance :

In many applications, the concept of the surface impedance is required. At the surface of the conductor, at higher frequencies, the total current gets totally confined to a thin sheet.

Def :

The impedance offered by the surface of a conductor (or) conducting medium is called surface impedance.

(or)

The surface impedance is defined as the ratio of the tangential component of the electric field to the surface current density at the conductor surface. It is denoted with Z_s and computed from the formula

$$\boxed{Z_s = \frac{E}{J_s} = \frac{\gamma}{\sigma}}$$

$J_s \rightarrow$ linear current density A/m

$\sigma \rightarrow$ conductivity

Proof :

If the plane wave is propagating towards z-direction, then the linear surface current density J_s is equal to

$$J_s = \int_0^\infty J e^{-\gamma z}\, dz = J\left[\frac{e^{-\gamma z}}{-\gamma}\right]_0^\infty$$

$$J_s = \frac{-J}{\gamma}\left[e^{-\infty z} - e^{-0}\right] = \frac{-J}{\gamma}(0-1)$$

$$J_s = \frac{J}{\gamma} = \frac{\sigma E}{\gamma}$$

$$\frac{J_s}{E} = \frac{\sigma}{\gamma} \Rightarrow \boxed{\frac{E}{J_s} = \frac{\gamma}{\sigma} = Z_s}$$

$\because \gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

$\eta = \sqrt{\frac{j\omega\mu}{\sigma}}$

→ For good conducting medium $\gamma = \sqrt{j\omega\mu\sigma}$

$$\boxed{\therefore Z_s = \frac{\gamma}{\sigma} = \frac{\sqrt{j\omega\mu\sigma}}{\sigma} = \sqrt{\frac{j\omega\mu}{\sigma}} = \eta}$$

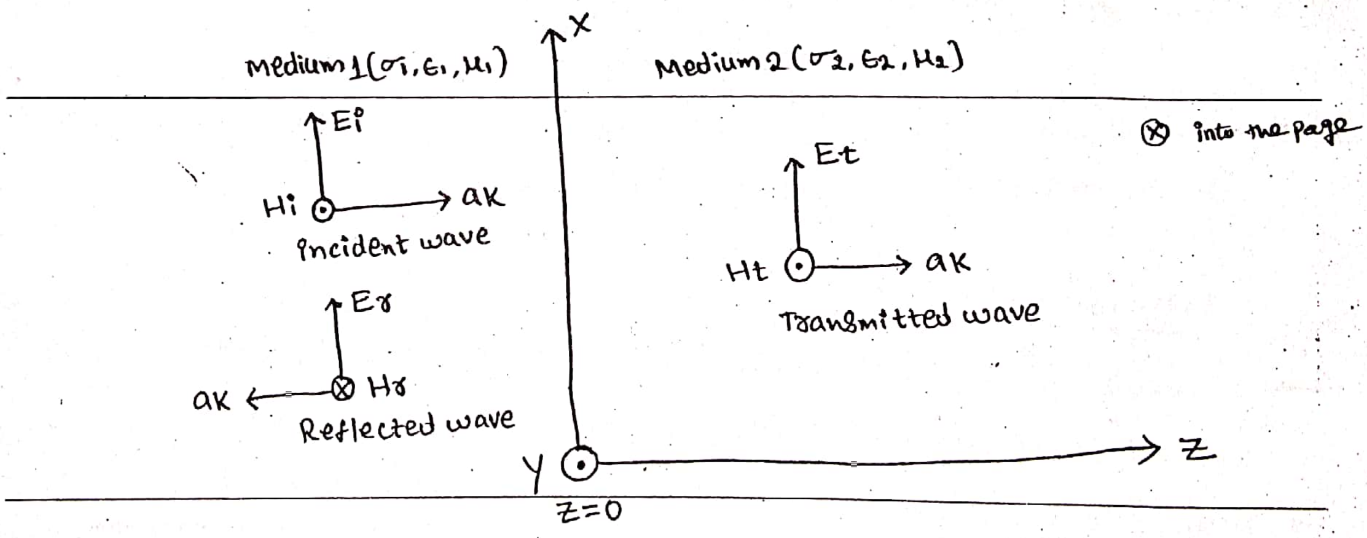
→ Hence from the above equation it is clear that for the good conductors, the surface impedance of a plane conductor with thickness greater than the skin depth of a conductor is equal to the characteristic impedance of the conductor. i.e. $\boxed{Z_s = \eta}$

Reflection of a plane wave at normal incidence :

When a plane wave from one medium meets a different medium, it is partly reflected and partly transmitted. The propagation of the incident wave that is reflected (or) transmitted depends on the constitutive parameters (σ, ϵ, μ) of the two media involved.

a). Perfect dielectric to Perfect dielectric :

Let us consider a plane wave is propagating along the +ve z-direction and it is incident normally on the boundary $z=0$ between medium 1 ($z<0$) characterised by $\sigma_1, \epsilon_1, \mu_1$ and medium 2 ($z>0$) characterised by $\sigma_2, \epsilon_2, \mu_2$ as shown in fig.



→ In figure, subscripts i, r and t denote incident, reflected and transmitted waves respectively.

→ When a plane wave is incident normally on the boundary then it is partly reflected and partly transmitted to the second medium.

→ The incident, reflected and transmitted waves are shown in fig. and obtained as follows.

Reflection

The ratio of

Incident wave :

→ In medium 1, the uniform plane wave is propagating along the +ve z-direction (or) towards z=0 is called incident wave.

→ If the direction of propagation of incident wave is normal to the boundary surface then it is called normal incidence.

let the incident wave $E_i(z) = E_{io}\, e^{-\gamma_1 z}\, a_x$

$$H_i(z) = H_{io}\, e^{-\gamma_1 z}\, a_y = \frac{E_{io}}{\eta_1} e^{-\gamma_1 z}\, a_y$$

Reflected wave :

→ In medium 1, the uniform plane wave moving away from the boundary (z=0) (or) towards -ve z-direction is called reflected wave.

→ Electric field and magnetic field components of reflected waves are

$$E_r(z) = E_{ro}\, e^{-\gamma_1(-z)}\, a_x = E_{ro}\, e^{\gamma_1 z}\, a_x$$

$$H_r(z) = H_{ro}\, e^{-\gamma_1(-z)}\, (-a_y) = -\frac{E_{ro}}{\eta_1} e^{\gamma_1 z}\, a_y$$

Transmitted wave :

→ In medium 2, the uniform plane wave moving away from the boundary (z=0) (or) towards +ve z-direction is called transmitted wave.

→ Electric field and magnetic field components of a transmitted waves are

$$E_t(z) = E_{to}\, e^{-\gamma_2 z}\, a_x$$

$$H_t(z) = H_{to}\, e^{-\gamma_2 z}\, a_y = \frac{E_{to}}{\eta_2} e^{-\gamma_2 z}\, a_y$$

Reflection & Transmission Coefficients :

The ratio between amplitude of reflected wave to amplitude of incident wave is called reflection coefficient. It is denoted with Γ and it can be computed from the formula

$$\boxed{\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}}$$

→ The magnitude of reflection coefficient lies in b/w 0 & 1 $\boxed{0 \le |\Gamma| \le 1}$

The ratio between amplitude of transmitted wave to amplitude of incident wave is called transmission coefficient. It is denoted with τ and computed from the formula

$$\boxed{\tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1}}$$

Proof:

Equate electric field & magnetic field components in medium 1 & medium 2.

$$E_i(z) + E_r(z) = E_t(z)$$

$$E_{io}\, e^{-\gamma_1 z} + E_{ro}\, e^{+\gamma_1 z} = E_{to}\, e^{-\gamma_2 z}$$

→ At the interface (or) on the boundary $z = 0$.

then, $E_{io} + E_{ro} = E_{to}$ ⟶ (1)

Similarly $H_i(z) + H_r(z) = H_t(z)$

$$\frac{E_{io}}{\eta_1} e^{-\gamma_1 z} - \frac{E_{ro}}{\eta_1} e^{\gamma_1 z} = \frac{E_{to}}{\eta_2} e^{-\gamma_2 z}$$

→ At the interface (or) on the boundary $z = 0$

then, $\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2}$

$E_{io} - E_{ro} = \frac{\eta_1}{\eta_2} E_{to}$ ⟶ (2)

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Reflection & Transmission Coefficients :

The ratio between amplitude of reflected wave to amplitude of incident wave is called reflection coefficient. It is denoted with Γ and it can be computed from the formula

$$\boxed{\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}}$$

→ The magnitude of reflection coefficient lies in b/w 0 & 1 $\boxed{0 \leq |\Gamma| \leq 1}$

The ratio between amplitude of transmitted wave to amplitude of incident wave is called transmission coefficient. It is denoted with τ and computed from the formula

$$\boxed{\tau = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_2 + \eta_1}}$$

Proof:

Equate electric field & magnetic field components in medium 1 & medium 2.

$$E_i(z) + E_r(z) = E_t(z)$$

$$E_{i0}\, e^{-\gamma_1 z} + E_{r0}\, e^{+\gamma_1 z} = E_{t0}\, e^{-\gamma_2 z}$$

→ At the interface (or) on the boundary $z = 0$.

then, $E_{i0} + E_{r0} = E_{t0}$ ⟶ (1)

Similarly $H_i(z) + H_r(z) = H_t(z)$

$$\frac{E_{i0}}{\eta_1} e^{-\gamma_1 z} - \frac{E_{r0}}{\eta_1} e^{\gamma_1 z} = \frac{E_{t0}}{\eta_2} e^{-\gamma_2 z}$$

→ At the interface (or) on the boundary $z = 0$

then, $\frac{E_{i0}}{\eta_1} - \frac{E_{r0}}{\eta_1} = \frac{E_{t0}}{\eta_2}$

$E_{i0} - E_{r0} = \frac{\eta_1}{\eta_2} E_{t0}$ ⟶ (2)

Dividing eq (1) with eq (2).

$$\frac{E_{io}+E_{ro}}{E_{io}-E_{ro}} = \frac{\cancel{E_{to}}}{\frac{\eta_1}{\eta_2}\cancel{E_{to}}} = \frac{\eta_2}{\eta_1}$$

$$\eta_1 E_{io} + \eta_1 E_{ro} = \eta_2 E_{io} - \eta_2 E_{ro}$$

$$E_{io}[\eta_1 - \eta_2] = -E_{ro}[\eta_2+\eta_1]$$

$$E_{ro}[\eta_2+\eta_1] = E_{io}[\eta_2-\eta_1]$$

$$\frac{E_{ro}}{E_{io}} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \Gamma$$

$$\boxed{\therefore \Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}}$$

Adding eq (1) and eq (2).

$$E_{io}+E_{ro}+E_{io}-E_{ro} = E_{to} + \frac{\eta_1}{\eta_2}E_{to} =$$

$$2E_{io} = E_{to}\left[1+\frac{\eta_1}{\eta_2}\right]$$

$$2E_{io} = E_{to}\left[\frac{\eta_2+\eta_1}{\eta_2}\right] \Rightarrow \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2+\eta_1}$$

$$\boxed{\tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2+\eta_1}}$$

Relation b/w Transmission coefficient & Reflection coefficient :

$$\tau - \Gamma = \frac{2\eta_2}{\eta_2+\eta_1} - \frac{(\eta_2-\eta_1)}{\eta_2+\eta_1} = \frac{2\eta_2-\eta_2+\eta_1}{\eta_2+\eta_1}$$

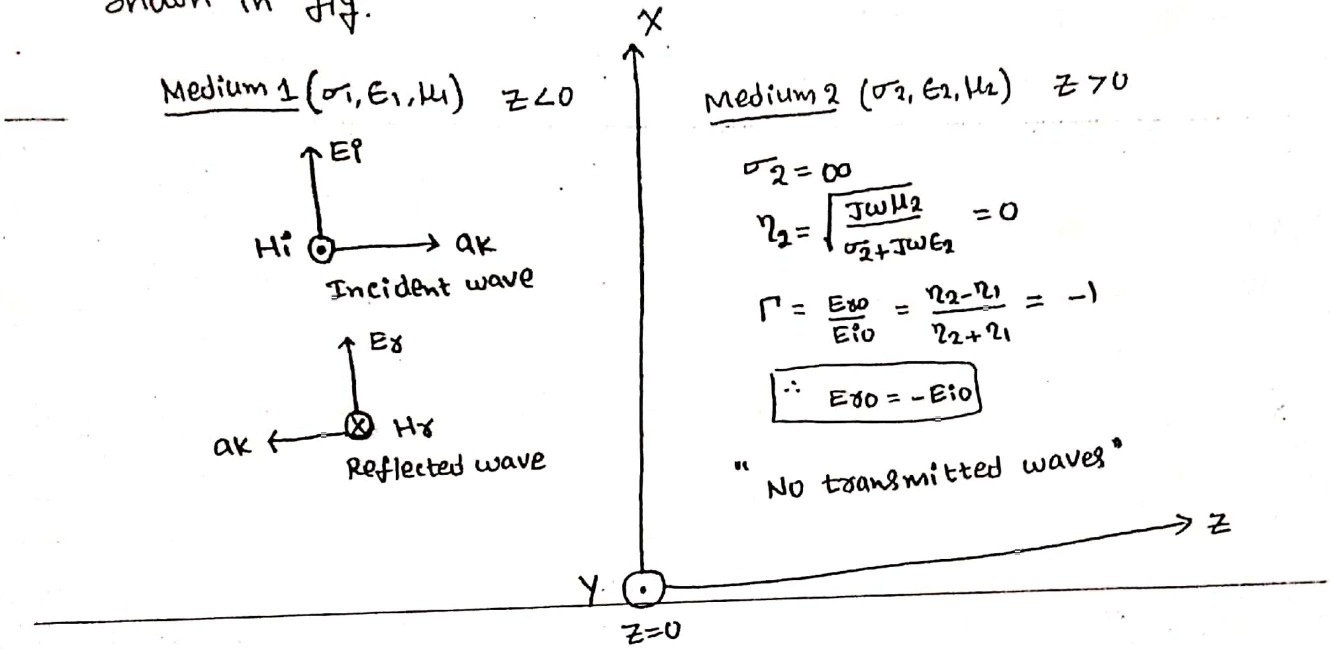
$$\tau - \Gamma = \frac{\eta_2+\eta_1}{\eta_2+\eta_1} = 1$$

$\boxed{\therefore \tau - \Gamma = 1}$ (or) $\boxed{1+\Gamma = \tau}$

→ Both Γ and τ are dimensionless and may be complex.

i) Perfect dielectric to Perfect Conductor :

Let us consider a plane wave is propagating along the +ve z-direction and it is incident normally on the boundary $z=0$ between perfect dielectric medium and perfect conducting medium as shown in fig.



Incident wave :

→ In medium 1, the uniform plane wave is propagating along the +ve z-direction (or) towards $z=0$ is called incident wave.

→ Electric field & magnetic field components of incident waves are

$$E_i(z) = E_{io}\, e^{-\gamma_1 z}\, a_x \; ; \quad H_i(z) = H_{io}\, e^{-\gamma_1 z}\, a_y = \frac{E_{io}}{\eta_1}\, e^{-\gamma_1 z}\, a_y$$

Reflected wave :

→ In medium 1, the uniform plane wave moving away from the boundary ($z=0$) (or) towards -ve z-direction is called reflected wave.

→ Electric field & magnetic field components of reflected waves are

$$E_r(z) = E_{ro}\, e^{-\gamma_1(-z)}\, a_x = E_{ro}\, e^{\gamma_1 z}\, a_x$$

$$H_r(z) = H_{ro}\, e^{-\gamma_1(-z)}\, (-a_y) = -\frac{E_{ro}}{\eta_1}\, e^{\gamma_1 z}\, a_y$$

Transmitted wave :

→ There is "no transmission".

→ For medium 2 (Perfect conductor), $\sigma = \infty$, intrinsic impedance $\eta_2 = 0$.

Thus the transmission coefficient is given by

$$\tau = \frac{2\eta_2}{\eta_2+\eta_1} = 0.$$

And the reflection coefficient is given by

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = -1$$

→ From the values of reflection & transmission coefficients it is clear that the wave is totally reflected and there is no transmitted wave in medium 2.

→ * It is obvious from the fact that no field can exists in the Perfect conductor. As totally reflected wave combines with the incident wave, standing waves are formed.

Standing wave :

If the plane wave is incident on the boundary of a conducting media, then the total wave is reflected back and produces a combination wave in medium 1 is called Standing wave.

→ A standing wave simply "stands" and doesnot travel. It consist of ~~two travelling waves namely incident wave & reflected wave. These~~ waves are equal in magnitude but in opposit direction.

→ Electric field components of a standing wave in medium 1 is

$$E_1(z) = E_i(z) + E_r(z)$$

$$E_1(z) = E_{i0}\, e^{-\gamma_1 z}\, a_x + E_{r0}\, e^{+\gamma_1 z}\, a_x$$

In medium 1 (Perfect dielectrics) $\alpha_1 = 0$; $\gamma_1 = j\beta_1$ and

$$\Gamma = \frac{E_{r0}}{E_{i0}} = -1 \Rightarrow E_{r0} = -E_{i0}$$

$$\frac{e^{J\theta} - e^{-J\theta}}{2} = J\sin\theta$$

$$\frac{e^{J\theta} + e^{-J\theta}}{2} = \cos\theta$$

$$e^{J\theta} = \cos\theta + J\sin\theta$$

..en

$\therefore$ $E_1(z) = E_{io}\, e^{-J\beta_1 z}\, a_x - E_{io}\, e^{J\beta_1 z}\, a_x$

$= -E_{io}\left[e^{J\beta_1 z} - e^{-J\beta_1 z}\right] a_x$

$= -E_{io}\left[2J\sin\beta_1 z\right] a_x$

$\therefore$ $E_1(z) = -2J\, E_{io}\sin(\beta_1 z)\, a_x$

$E_1(z,t) = \text{Real Part}\left[E_1(z)\, e^{J\omega_1 t}\right]$

$E_1(z,t) = \text{Re}\left[-J2\, E_{io}\sin(\beta_1 z)\left(\cos\omega_1 t + J\sin\omega_1 t\right)\right] a_x$

$$\boxed{E_1(z,t) = 2E_{io}\sin\beta_1 z\,\sin\omega_1 t\, a_x} \longrightarrow (1)$$

Similarly the magnetic field component of a standing wave in medium-1 is

$H_1(z) = H_i(z) + H_r(z)$

$= \frac{E_{io}}{\eta_1} e^{-\gamma_1 z}\, a_y - \frac{\cancel{E_{ro}}}{\eta_1} e^{\gamma_1 z}\, a_y$ $\quad$ $\therefore$ ~~$E_{ro} = -E_{io}$~~

$= \frac{E_{io}}{\eta_1} e^{-J\beta_1 z}\, a_y + \frac{E_{io}}{\eta_1} e^{J\beta_1 z}\, a_y$

$= \frac{E_{io}}{\eta_1}\left[e^{J\beta_1 z} + e^{-J\beta_1 z}\right] a_y$

$= \frac{E_{io}}{\eta_1}\left[2\cos\beta_1 z\right] a_y$

$\therefore$ $H_1(z) = \frac{2E_{io}}{\eta_1}\cos\beta_1 z\, a_y$

$\therefore$ $H_1(z,t) = \text{Real Part}\left[H_1(z)\, e^{J\omega_1 t}\right]$

$H_1(z,t) = \text{Re}\left[\frac{2E_{io}}{\eta_1}\cos\beta_1 z\left(\cos\omega_1 t + J\sin\omega_1 t\right)\right]$

$$\boxed{H_1(z,t) = \frac{2E_{io}}{\eta_1}\cos\beta_1 z\,\cos\omega_1 t\, a_y} \longrightarrow (2)$$

→ eq (1) & (2) shows that the incident and reflected waves combines to produce a standing wave.

Standing Wave Ratio (SWR) :

→ When the incident and reflected waves are in phase (0° (or) 360°), we get the maximum amplitude of the field

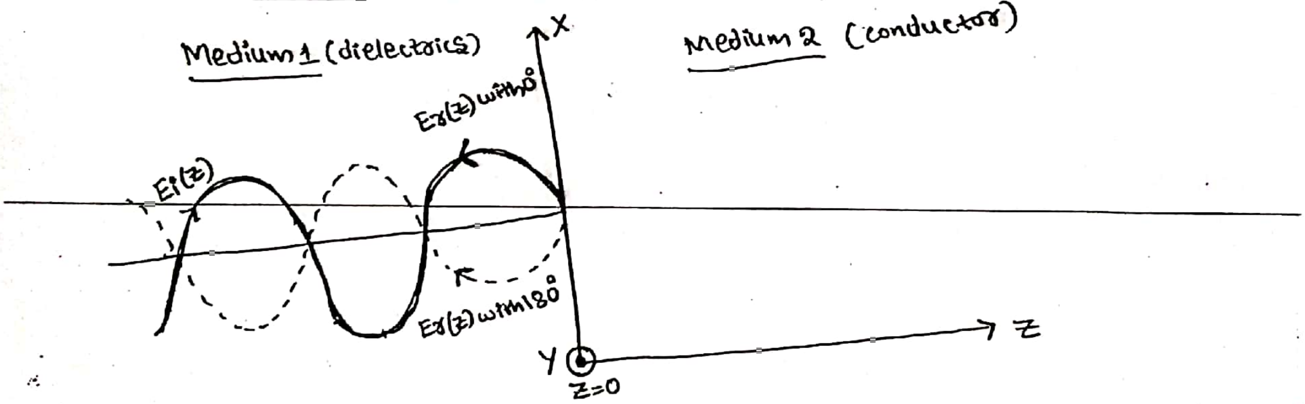
$$\boxed{\therefore E_{1max} = (1+|\Gamma|)E_{io}}$$

→ When the incident and reflected waves are out of phase by 180°, we get the minimum amplitude of the field.

$$\boxed{\therefore E_{1min} = (1-|\Gamma|)E_{io}}$$

The standing wave ratio (S) is defined as the ratio of maximum to minimum amplitudes of voltage

$$\boxed{\therefore S = \frac{E_{1max}}{E_{1min}} = \frac{H_{1max}}{H_{1min}} = \frac{1+|\Gamma|}{1-|\Gamma|}}$$



$E_1(z) = E_i(z) + E_r(z)$

$E_{1max} = |E_{io}| + |E_{ro}| = E_{io}\left[1 + \frac{|E_{ro}|}{|E_{io}|}\right] = E_{io}[1+|\Gamma|]$

$E_{1min} = |E_{io}| - |E_{ro}| = E_{io}\left[1 - \frac{|E_{ro}|}{|E_{io}|}\right] = E_{io}[1-|\Gamma|]$

$$\boxed{\therefore S = \frac{E_{1max}}{E_{1min}} = \frac{E_{io}[1+|\Gamma|]}{E_{io}[1-|\Gamma|]} = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

→ When the amplitude of incident & reflected waves are equal, then $|\Gamma| = 1$ & $S = \infty$. It indicates total energy is reflected.

→ When there is no reflection of energy i.e., $\eta_2 = \eta_1$, then $|\Gamma| = 0$ & $S = 1$.

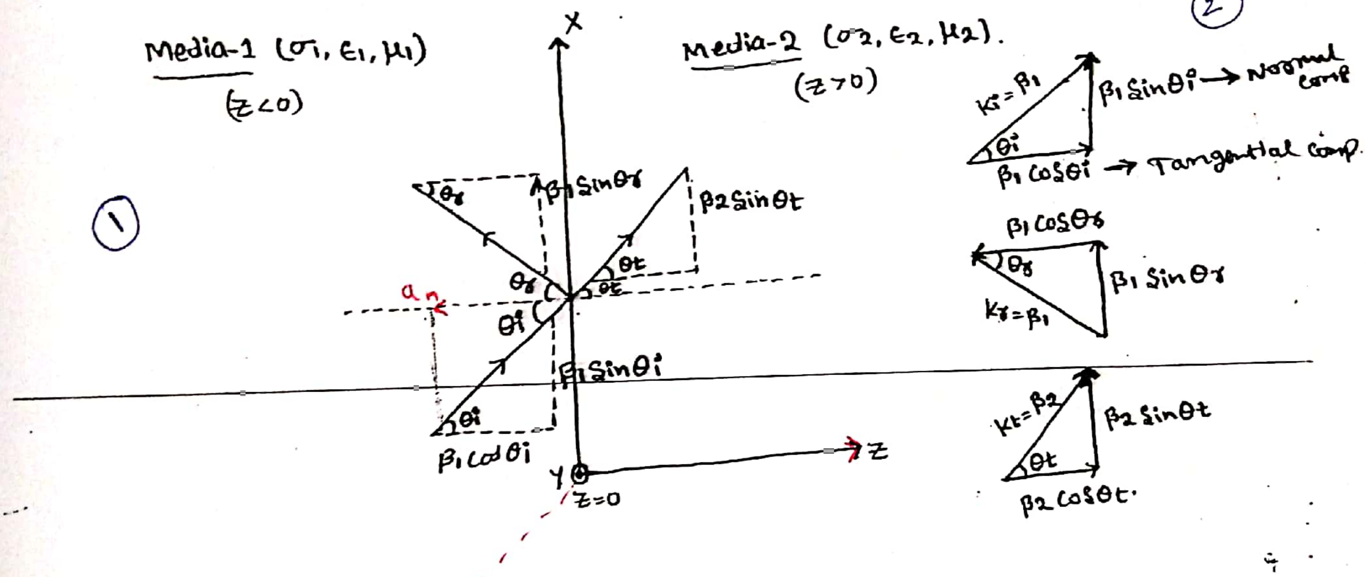
→ The standing wave ratio 'S' is dimensionless & its value lies in the range $\boxed{1 \leq S \leq \infty}$

→ We can express 'Γ' in terms of 'S' as $\boxed{|\Gamma| = \frac{S-1}{S+1}}$

Reflection of a plane wave at oblique incidence :

When a plane wave is incident on the boundary surface $z=0$ between two different media with an angle of incidence (θ_i), $[0° < \theta_i < 90°]$ is called oblique incidence.

→ Let us consider oblique incidence of a plane wave on the boundary $(z=0)$ between medium-1 $(z<0)$ characterised by $(\sigma_1, \epsilon_1, \mu_1)$ and medium-2 $(z>0)$ characterised by $(\sigma_2, \epsilon_2, \mu_2)$. is shown in fig.



→ If a plane wave is incident with an angle of incidence (θ_i) then it is partly reflected back and partly transmitted to the second medium with an angle of reflection (θ_r) and transmitted angle (θ_t) respectively.

→ From the above 3 similar Δ^{les},

$$\beta_1 \sin\theta_i = \beta_1 \sin\theta_r = \beta_2 \sin\theta_t$$

$$\Rightarrow \beta_1 \sin\theta_i = \beta_1 \sin\theta_r \Rightarrow \sin\theta_i = \sin\theta_r \Rightarrow \boxed{\theta_i = \theta_r}$$ — Snells law

* From the above equation, it is clear that the angle of incidence (θ_i) is equal to the angle of reflection (θ_r). This is known as "Snell's law of reflection"

$$\Rightarrow \beta_1 \sin\theta_i = \beta_2 \sin\theta_t$$

$\beta = \omega\sqrt{\mu\epsilon}$

Refractive index

$n = \sqrt{\mu_r \epsilon_r}$

$$\omega\sqrt{\mu_1\epsilon_1}\sin\theta_i = \omega\sqrt{\mu_2\epsilon_2}\sin\theta_t$$

$$\sqrt{\mu_0\mu_{r1}\epsilon_0\epsilon_{r1}}\sin\theta_i = \sqrt{\mu_0\mu_{r2}\epsilon_0\epsilon_{r2}}\sin\theta_t$$

$$\sqrt{\mu_{r1}\epsilon_{r1}}\sin\theta_i = \sqrt{\mu_{r2}\epsilon_{r2}}\sin\theta_t$$

$$\boxed{n_1 \sin\theta_i = n_2 \sin\theta_t}$$

* The above equation is known as "Snell's law of refraction".
It gives the relation b/w angle of incidence (θ_i) & angle of transmission (θ_t).

→ There are two cases for oblique incidence as given below.

a) Parallel Polarization :

$E \parallel^{lel}$ boundary $- H$ (or) $\perp^{ler}$

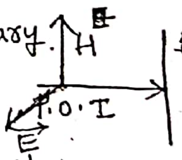
$E \parallel$ plane → V (or) para

Polarisation:

The time varying behavior of an electric field strength vector E. at some fixed point in a space is called Polarization.

Plane of incidence :

It is a plane which contains the incident, reflected and transmitted rays and is normal to the boundary.



Def :

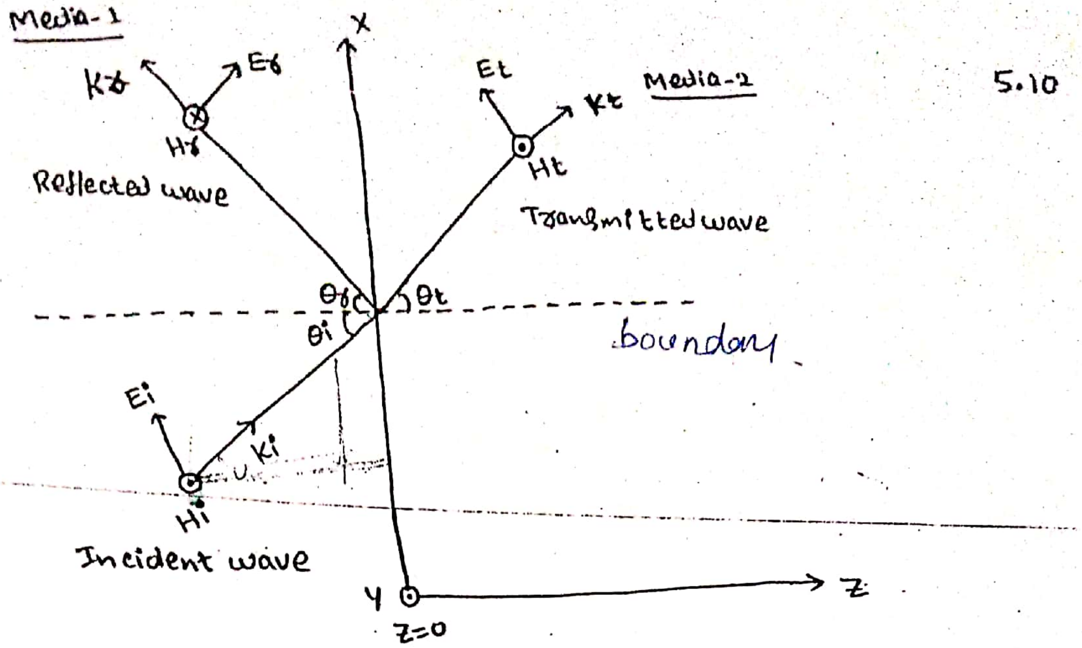
It is defined as the polarisation in which the electric field of the wave is parallel to the plane of incidence.

(or)

It is defined as the polarisation in which the magnetic field vector is perpendicular to the plane of incidence.

→ Parallel polarisation is also called vertical polarisation

plane of Incidence :- The plane defined by the propagation vector K and a unit normal vector an to the boundary is called plane of incidence.



Reflection and Transmission coefficients:

Equate tangential components in medium-1 and medium-2

$$(E_i)_{tan} + (E_r)_{tan} = (E_t)_{tan}$$

$$E_{io}\cos\theta_i + E_{ro}\cos\theta_r = E_{to}\cos\theta_t$$

$$E_{io}\cos\theta_i + E_{ro}\cos\theta_i = E_{to}\cos\theta_t \qquad \because \theta_i = \theta_r$$

$$E_{io} + E_{ro} = \frac{\cos\theta_t}{\cos\theta_i} E_{to} \longrightarrow (1)$$

Similarly,

$$(H_i)_{tan} + (H_r)_{tan} = (H_t)_{tan}$$

$$\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2}$$

$$E_{io} - E_{ro} = \frac{\eta_1}{\eta_2} E_{to} \longrightarrow (2)$$

Dividing eq(1) with eq(2)

$$\frac{(1)}{(2)} \Rightarrow \frac{E_{io} + E_{ro}}{E_{io} - E_{ro}} = \frac{\cos\theta_t}{\cos\theta_i} \times \frac{\eta_2}{\eta_1}$$

By using componendo & dividendo rule

$$\frac{E_{io}+\cancel{E_{ro}}+E_{io}-\cancel{E_{ro}}}{\cancel{E_{io}}+E_{ro}-\cancel{E_{io}}+E_{ro}} = \frac{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}$$

$$\frac{2E_{io}}{2E_{ro}} = \frac{E_{io}}{E_{ro}} = \frac{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}$$

$$\boxed{\therefore\ \Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}} \longrightarrow (3) ✓$$

→ Adding eq (1) + eq (2)

$$E_{io}+\cancel{E_{ro}}+E_{io}-\cancel{E_{ro}} = \frac{\cos\theta_t}{\cos\theta_i} E_{to} + \frac{\eta_1}{\eta_2} E_{to}$$

$$2E_{io} = E_{to}\left[\frac{\cos\theta_t}{\cos\theta_i} + \frac{\eta_1}{\eta_2}\right]$$

$$2E_{io} = E_{to}\left[\frac{\eta_2\cos\theta_t + \eta_1\cos\theta_i}{\cos\theta_i \cdot \eta_2}\right]$$

$$\boxed{\frac{E_{to}}{E_{io}} = \tau = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}} \longrightarrow (4) ✓$$

→ The above equations (3) & (4) are called Fresnel's equations.

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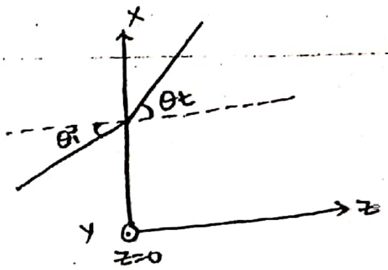
Brewster angle :

It is the angle of incidence at which there is no reflection is called Brewster angle.

(or)

It is the angle of incidence at which the complete incident wave will be transmitted to the 2^{nd} region without reflecting back into medium 1. It is denoted with θ_B and for a lossless medium

$$\boxed{\theta_B = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}} \quad \text{as } \mu_1 = \mu_2$$



Proof:

If $\theta_i = \theta_B$ then $\Gamma = \frac{E_{ro}}{E_{io}} = 0$

→ from eq (3)
$$\frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = 0$$

$$\eta_2 \cos\theta_t - \eta_1 \cos\theta_B = 0 \qquad \because \theta_i = \theta_B$$

$$\eta_2 \cos\theta_t = \eta_1 \cos\theta_B$$

$$\cos\theta_t = \frac{\eta_1}{\eta_2}\cos\theta_B \longrightarrow (5)$$

→ from Snell's law of refraction

$$\Rightarrow n_1 \sin\theta_i = n_2 \sin\theta_t \Rightarrow n_1 \sin\theta_B = n_2 \sin\theta_t$$

$$\Rightarrow \sin\theta_B = \frac{n_2}{n_1}\sin\theta_t \Rightarrow \sin^2\theta_B = \left(\frac{n_2}{n_1}\right)^2 \sin^2\theta_t$$

$$\Rightarrow \sin^2\theta_B = \left(\frac{n_2}{n_1}\right)^2\left[1 - \cos^2\theta_t\right] \longrightarrow (6)$$

→ Substitute eq (5) in eq (6)

$$\sin^2\theta_B = \left(\frac{n_2}{n_1}\right)^2\left[1 - \left(\frac{\eta_1}{\eta_2}\right)^2 \cos^2\theta_B\right]$$

$$= \left(\frac{n_2}{n_1}\right)^2\left[1 - \left(\frac{\eta_1}{\eta_2}\right)^2 (1 - \sin^2\theta_B)\right]$$

$$= \left(\frac{n_2}{n_1}\right)^2\left[1 - \left(\frac{\eta_1}{\eta_2}\right)^2 + \left(\frac{\eta_1}{\eta_2}\right)^2 \sin^2\theta_B\right]$$

$$\sin^2\theta_B - \left(\frac{n_2}{n_1}\right)^2\left(\frac{\eta_1}{\eta_2}\right)^2\sin^2\theta_B = \left(\frac{n_2}{n_1}\right)^2\left[1-\left(\frac{\eta_1}{\eta_2}\right)^2\right]$$

$$\sin^2\theta_B\left[1-\left(\frac{n_2}{n_1}\right)^2\left(\frac{\eta_1}{\eta_2}\right)^2\right] = \left(\frac{n_2}{n_1}\right)^2\left[1-\left(\frac{\eta_1}{\eta_2}\right)^2\right]$$

$$\sin^2\theta_B = \frac{\left(\frac{n_2}{n_1}\right)^2\left[1-\left(\frac{\eta_1}{\eta_2}\right)^2\right]}{\left[1-\left(\frac{n_2}{n_1}\right)^2\left(\frac{\eta_1}{\eta_2}\right)^2\right]} \longrightarrow (7)$$

→ for loss less medium $\frac{\sigma}{\omega\epsilon} << 1$; $\eta = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\epsilon}} = \sqrt{\frac{\mu}{\epsilon}}$

$$\begin{aligned}\eta_1 &= \sqrt{\frac{\mu_1}{\epsilon_1}}\\ \eta_2 &= \sqrt{\frac{\mu_2}{\epsilon_2}}\end{aligned} \Rightarrow \frac{\eta_1}{\eta_2} = \sqrt{\frac{\mu_1\epsilon_2}{\mu_2\epsilon_1}}$$

; Refractive Index $n=\sqrt{\mu_r\epsilon_r}$ $\Rightarrow \frac{n_2}{n_1} = \sqrt{\frac{\mu_{r2}\epsilon_{r2}}{\mu_{r1}\epsilon_{r1}}} = \sqrt{\frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}}$

then (7) ⇒ $$\sin^2\theta_B = \frac{\frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}\left[1-\frac{\mu_1\epsilon_2}{\mu_2\epsilon_1}\right]}{1-\left(\frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}\right)\left(\frac{\mu_1\epsilon_2}{\mu_2\epsilon_1}\right)}$$

$$\cancel{\sin^2\theta_B = \frac{\frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}-\left(\frac{\epsilon_2}{\epsilon_1}\right)^2}{1-\left(\frac{\epsilon_2}{\epsilon_1}\right)^2}}$$

→ Given $\mu_1=\mu_2$ then $$\sin^2\theta_B = \frac{\frac{\epsilon_2}{\epsilon_1}-\left(\frac{\epsilon_2}{\epsilon_1}\right)^2}{1-\left(\frac{\epsilon_2}{\epsilon_1}\right)^2} = \frac{\frac{\epsilon_2}{\epsilon_1}\left[1-\cancel{\frac{\epsilon_2}{\epsilon_1}}\right]}{\left(1-\cancel{\frac{\epsilon_2}{\epsilon_1}}\right)\left(1+\frac{\epsilon_2}{\epsilon_1}\right)}$$

$$\sin^2\theta_B = \frac{\frac{\epsilon_2}{\epsilon_1}}{1+\frac{\epsilon_2}{\epsilon_1}} = \frac{\epsilon_2/\cancel{\epsilon_1}}{\frac{\epsilon_1+\epsilon_2}{\cancel{\epsilon_1}}} = \frac{\epsilon_2}{\epsilon_1+\epsilon_2}$$

$$\sin\theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1+\epsilon_2}} \Rightarrow \theta_B = \sin^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1+\epsilon_2}}$$

from Δ^{le} $$a^2 = \sqrt{\epsilon_1+\epsilon_2}^{\,2} - \sqrt{\epsilon_2}^{\,2}$$

$$a^2 = \epsilon_1+\cancel{\epsilon_2}-\cancel{\epsilon_2} = \epsilon_1$$

$$\boxed{a=\sqrt{\epsilon_1}}$$

$$\therefore \tan\theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \Rightarrow \boxed{\theta_B = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

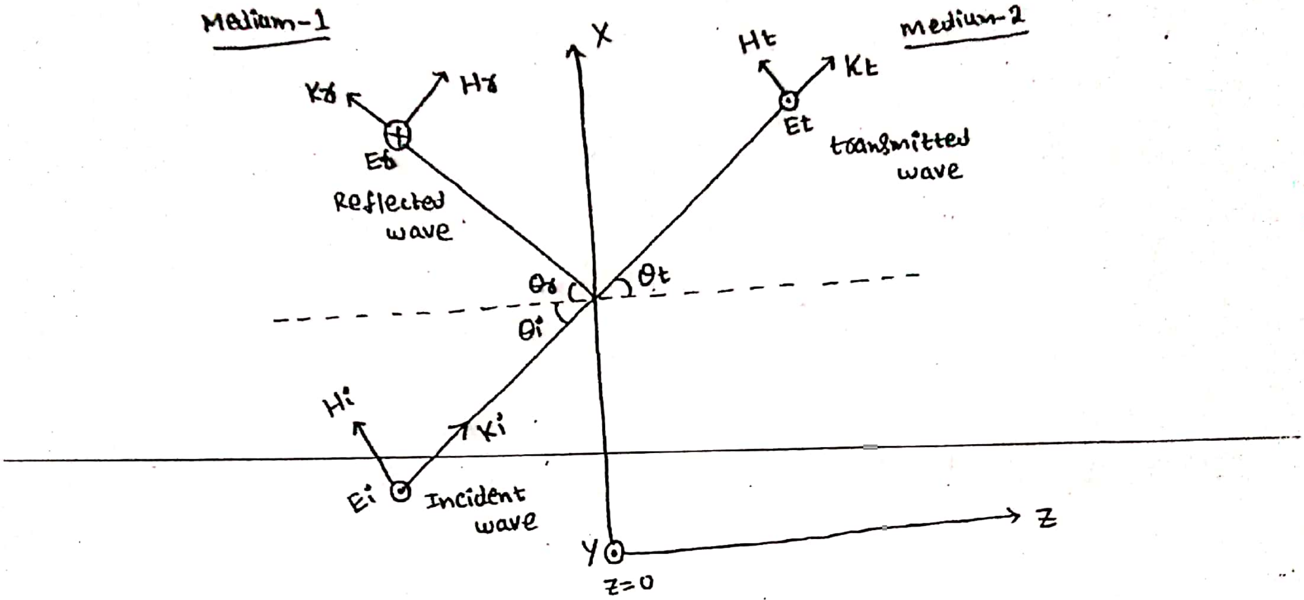
ii) Perpendicular Polarisation :

Def: It is defined as the polarisation in which the electric field of the wave is perpendicular to the plane of incidence.

(or)

It is defined as the polarisation in which the magnetic field vector is parallel to the plane of incidence.

→ Perpendicular polarisation is also called horizontal polarisation.



Reflection and Transmission coefficients :

Equate tangential components in medium-1 and medium-2.

$$(H_i)_{tan} + (H_r)_{tan} = (H_t)_{tan}$$

$$H_{io}\cos\theta_i + H_{ro}\cos\theta_r = H_{to}\cos\theta_t$$

$$H_{io}\cos\theta_i + H_{ro}\cos\theta_i = H_{to}\cos\theta_t \qquad \because \theta_i = \theta_r$$

$$\cos\theta_i\,[H_{io} + H_{ro}] = H_{to}\cos\theta_t$$

$$\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2}\,\frac{\cos\theta_t}{\cos\theta_i}$$

$$E_{io} - E_{ro} = \frac{\eta_1}{\eta_2}\,\frac{\cos\theta_t}{\cos\theta_i}\,E_{to} \longrightarrow (1)$$

Similarly $E_{io} + E_{ro} = E_{to} \longrightarrow (2)$

Dividing eq (2) with eq (1)

$$\frac{(2)}{(1)} \Rightarrow \frac{E_{io} + E_{ro}}{E_{io} - E_{ro}} = \frac{E_{to}}{\frac{\eta_1}{\eta_2}\frac{\cos\theta_t}{\cos\theta_i} E_{to}} = \frac{\eta_2 \cos\theta_i}{\eta_1 \cos\theta_t}$$

By using componendo & dividendo rule

$$\frac{E_{io} + \cancel{E_{ro}} + E_{io} - \cancel{E_{ro}}}{\cancel{E_{io}} + E_{ro} - \cancel{E_{io}} + E_{ro}} = \frac{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}$$

$$\frac{2E_{io}}{2E_{ro}} = \frac{E_{io}}{E_{ro}} = \frac{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}$$

$$\boxed{\therefore \Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}} \longrightarrow (3)$$

→ Adding eq (1) & eq (2)

$$(1)+(2) \quad E_{io} - \cancel{E_{ro}} + E_{io} + \cancel{E_{ro}} = \frac{\eta_1}{\eta_2}\frac{\cos\theta_t}{\cos\theta_i} E_{to} + E_{to}$$

$$2E_{io} = E_{to}\left[1 + \frac{\eta_1}{\eta_2}\frac{\cos\theta_t}{\cos\theta_i}\right]$$

$$2E_{io} = E_{to}\left(\frac{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i}\right)$$

$$\frac{E_{to}}{E_{io}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\boxed{\therefore \tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}} \longrightarrow (4).$$

Brewster angle :

It is the angle of incidence at which there is no reflection is called Brewster angle.

$$\boxed{\theta_B = \tan^{-1}\sqrt{\frac{\mu_{r2}}{\mu_{r1}}}} \quad \text{as } \epsilon_{r1} = \epsilon_{r2}$$

Proof: If $\theta_i = \theta_B$ then $\Gamma = \frac{E_{ro}}{E_{io}} = 0$

→ from eq (3) $\quad \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = 0$

$\because \theta_i = \theta_B$

$$\eta_2 \cos\theta_B - \eta_1 \cos\theta_t = 0$$

$$\eta_2 \cos\theta_B = \eta_1 \cos\theta_t$$

$$\cos\theta_t = \frac{\eta_2}{\eta_1} \cos\theta_B \longrightarrow (5)$$

→ from Snell's law of refraction

$\Rightarrow n_1 \sin\theta_i = n_2 \sin\theta_t \Rightarrow n_1 \sin\theta_B = n_2 \sin\theta_t$

$\Rightarrow \sin\theta_B = \frac{n_2}{n_1} \sin\theta_t \Rightarrow \sin^2\theta_B = \left(\frac{n_2}{n_1}\right)^2 \sin^2\theta_t$

$\Rightarrow \sin^2\theta_B = \left(\frac{n_2}{n_1}\right)^2 \left[1 - \cos^2\theta_t\right] \longrightarrow (6)$

→ Substitute eq (5) in eq (6).

$$\sin^2\theta_B = \left(\frac{n_2}{n_1}\right)^2 \left[1 - \left(\frac{\eta_2}{\eta_1}\right)^2 \cos^2\theta_B\right]$$

$$= \left(\frac{n_2}{n_1}\right)^2 \left[1 - \left(\frac{\eta_2}{\eta_1}\right)^2 (1 - \sin^2\theta_B)\right]$$

$$= \left(\frac{n_2}{n_1}\right)^2 \left[1 - \left(\frac{\eta_2}{\eta_1}\right)^2 + \left(\frac{\eta_2}{\eta_1}\right)^2 \sin^2\theta_B\right]$$

$$\sin^2\theta_B - \left(\frac{n_2}{n_1}\right)^2 \left(\frac{\eta_2}{\eta_1}\right)^2 \sin^2\theta_B = \left(\frac{n_2}{n_1}\right)^2 \left[1 - \left(\frac{\eta_2}{\eta_1}\right)^2\right]$$

$$\sin^2\theta_B \left[1 - \left(\frac{n_2}{n_1}\right)^2 \left(\frac{\eta_2}{\eta_1}\right)^2\right] = \left(\frac{n_2}{n_1}\right)^2 \left[1 - \left(\frac{\eta_2}{\eta_1}\right)^2\right]$$

$$\sin^2\theta_B = \frac{\left(\frac{n_2}{n_1}\right)^2\left[1-\left(\frac{\eta_2}{\eta_1}\right)^2\right]}{\left[1-\left(\frac{n_2}{n_1}\right)^2\left(\frac{\eta_2}{\eta_1}\right)^2\right]} \longrightarrow (7)$$

$$\sin^2\theta_B = \frac{\left(\frac{n_2}{n_1}\right)^2-\left(\frac{n_2}{n_1}\right)^2\left(\frac{\eta_2}{\eta_1}\right)^2}{1-\left(\frac{n_2}{n_1}\right)^2\left(\frac{\eta_2}{\eta_1}\right)^2} \longrightarrow (7).$$

→ But we know that $\eta = \sqrt{\frac{\mu}{\epsilon}} \Rightarrow \frac{\eta_2}{\eta_1} = \sqrt{\frac{\mu_2\epsilon_1}{\mu_1\epsilon_2}} = \sqrt{\frac{\mu_{r2}\epsilon_{r1}}{\mu_{r1}\epsilon_{r2}}}$

and $n = \sqrt{\mu_r\epsilon_r} \Rightarrow \frac{n_2}{n_1} = \sqrt{\frac{\mu_{r2}\epsilon_{r2}}{\mu_{r1}\epsilon_{r1}}}$

then from eq (7)

$$\sin^2\theta_B = \frac{\frac{\mu_{r2}\epsilon_{r2}}{\mu_{r1}\epsilon_{r1}} - \frac{\mu_{r2}\cancel{\epsilon_{r2}}}{\mu_{r1}\cancel{\epsilon_{r1}}}\times\frac{\mu_{r2}\cancel{\epsilon_{r1}}}{\mu_{r1}\cancel{\epsilon_{r2}}}}{1-\frac{\mu_{r2}\cancel{\epsilon_{r2}}}{\mu_{r1}\cancel{\epsilon_{r1}}}\times\frac{\mu_{r2}\cancel{\epsilon_{r1}}}{\mu_{r1}\cancel{\epsilon_{r2}}}}$$

$$\sin^2\theta_B = \frac{\frac{\mu_{r2}\epsilon_{r2}}{\mu_{r1}\epsilon_{r1}} - \frac{\mu_{r2}^2}{\mu_{r1}^2}}{1-\frac{\mu_{r2}^2}{\mu_{r1}^2}} = \frac{\frac{\mu_{r2}}{\mu_{r1}}\left[\frac{\epsilon_{r2}}{\epsilon_{r1}} - \frac{\mu_{r2}}{\mu_{r1}}\right]}{1-\frac{\mu_{r2}^2}{\mu_{r1}^2}}$$

If $\epsilon_{r1} = \epsilon_{r2}$;

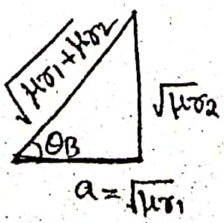
then $$\sin^2\theta_B = \frac{\frac{\mu_{r2}}{\mu_{r1}}\left(1-\frac{\mu_{r2}}{\mu_{r1}}\right)}{\left(1-\frac{\mu_{r2}}{\mu_{r1}}\right)\left(1+\frac{\mu_{r2}}{\mu_{r1}}\right)} = \frac{\frac{\mu_{r2}}{\mu_{r1}}}{1+\frac{\mu_{r2}}{\mu_{r1}}}$$

$$\sin^2\theta_B = \frac{\mu_{r2}}{\mu_{r1}+\mu_{r2}} \Rightarrow \sin\theta_B = \sqrt{\frac{\mu_{r2}}{\mu_{r1}+\mu_{r2}}}$$

from Δ^{le} $a^2 = \sqrt{\mu_{r1}+\mu_{r2}}^{\,2} - \sqrt{\mu_{r2}}^{\,2}$

$a^2 = \mu_{r1} + \cancel{\mu_{r2}} - \cancel{\mu_{r2}} = \mu_{r1}$

$\boxed{a = \sqrt{\mu_{r1}}}$



$$\therefore \tan\theta_B = \sqrt{\frac{\mu_{r2}}{\mu_{r1}}} \Rightarrow \boxed{\theta_B = \tan^{-1}\sqrt{\frac{\mu_{r2}}{\mu_{r1}}}}$$

Critical angle and Total internal reflection :

Def : It is the angle of incidence at which there is no transmission is called critical angle

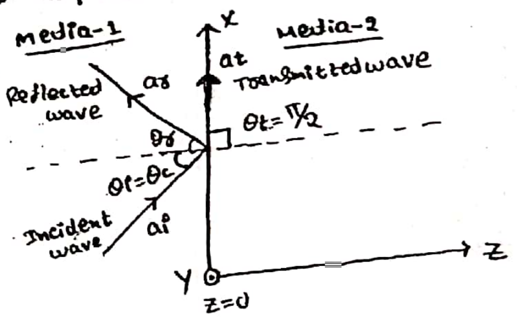
(or)

It is an angle of incidence at which the complete incident wave (or) power will be reflected back without transmitting into the next region.

→ It is denoted with θ_c and it can be computed from the formula

$$\boxed{\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

where n_1 & n_2 are refractive indices



→ Consider that medium-1 is denser as compared to medium-2 ie, $\epsilon_1 > \epsilon_2$. Under this condition the angle of transmission θ_t becomes greater than the angle of incidence θ_i.

→ The angle of transmission θ_t increases with angle of incidence θ_i. When $\theta_t = \frac{\pi}{2}$, the transmitted wave will be aligned along the interface as shown in fig

T.I.R → If θ_i is increases further, then there will be no transmitted wave and the condition is named as total reflection.

Proof : from Snell's law of refraction,

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

If $\theta_i = \theta_c$, then $\theta_t = 90^\circ$ (or) $\pi/2$.

then $n_1 \sin\theta_c = n_2 \sin 90$

$$n_1 \sin\theta_c = n_2$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

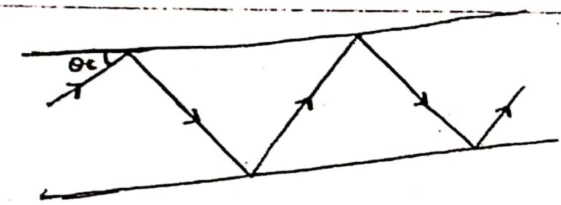
$$\boxed{\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

Def:

The process of reflecting total incident wave (or) Power without transmission is called total reflection (or) total internal reflection.

→ The condition for total internal reflection is $\boxed{\theta_i \geq \theta_c}$

Ex wave propagating through optical fiber



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(7) In a nonmagnetic medium $E = 4\sin(2\pi\times10^7 t - 0.8x)\,a_z$ V/m.

find (a) ϵ_r, η

(b) The time average power carried by the wave

(c) The total power crossing 100 cm² of plane $2x+y=5$

Sol

Give $E = 4\sin(2\pi\times10^7 t - 0.8x)\,a_z$ V/m

$E_0 = 4$, $\omega = 2\pi\times10^7$, $\beta = 0.8$

In non magnetic $\mu = \mu_0$ & $\epsilon = \epsilon_0\epsilon_r$.

a) $\beta = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0\epsilon_0\epsilon_r} = \frac{\omega}{c}\sqrt{\epsilon_r}$.

$$\sqrt{\epsilon_r} = \frac{\beta c}{\omega} = \frac{0.8(3\times10^8)}{2\pi\times10^7} = \frac{12}{\pi} \Rightarrow \epsilon_r = 14.59.$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0\epsilon_r}} = \frac{120\pi}{\sqrt{\epsilon_r}} = \frac{120\pi}{12/\pi} = 10\pi^2 = 98.7\,\Omega$$

b) Time avg power $P_{avg} = \frac{E_0^2}{2|\eta|}e^{-2\alpha z}\cos\theta_\eta\, a_z$ [(Lossy medium) wave z-direction in general].

for dielectric $\alpha=0$; $\theta_\eta = 0$ then

$$P_{avg} = \frac{E_0^2}{2|\eta|}a_x = \frac{(4)^2}{2(98.7)}a_x = 81\, a_x \text{ mW/m}^2$$

c) plane is on $2x+y=5$; surface area $= 100$ cm²

~~Vector = $2a_x + a_y$~~

Unit normal vector $a_n = \frac{2a_x + a_y}{\sqrt{2^2+1^2}} = \frac{2a_x+a_y}{\sqrt5}$.

~~total power~~ $= \int P_{avg}\cdot ds = P_{avg}\cdot S\, a_n$

$$= (81\times10^{-3}a_x)\cdot(100\times10^{-4})\left[\frac{2a_x+a_y}{\sqrt5}\right]$$

$$= \frac{81\times2\times10^{-5}}{\sqrt5}(a_x\cdot a_x)$$

$$= \frac{162\times10^{-5}}{\sqrt5}(1).$$

$$= 724.5\ \mu W.$$

P_{avg} = Power density

Power = $\int$ Power density

Power density = $\frac{d}{dt}$ Power